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Microwave interactions with low energy electrons

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Abstract. The equations of motion for a low energy electron interacting with a microwave electric field at the electron cyclotron frequency in a homogeneous magnetic field are solved numerically to determine the electron energy as a function of time and microwave electric field strength.

The shape of the microwave power absorption signal is calculated for two ideal cases and for more practical cases in which both the interaction time and the mean time between collisions must be considered.

1. Introduction

The solution of the equations of motion of an electron gyrating in a magnetic field in the presence of a microwave field has proved difficult except for special cases. The problem is simplified if low energy electrons are considered. Consoli and Mourier (1963) have calculated the maximum energy attained by electrons, initially at rest, which encounter a microwave field at the electron cyclotron resonance frequency, although they do not determine the electron energy as a function of time. According to Hakkenburg and Weenink (1964) higher energies are attained if the microwave frequency is less than the electron cyclotron frequency. The energy gain of charged particles interacting with two high frequency waves beating at the gyroresonance frequency has been investigated by Crescentini *et al* (1971). The first part of this present paper shows some results obtained by computer solution of the equations of motion for low energy electrons.

The power absorbed by relativistic electrons undergoing cyclotron resonance has been discussed by several authors in connection with electron cyclotron masers (Hirshfield *et al* 1965, Hsu and Robson 1965). Franken and Liebes (1959) and Klein (1968) discuss the power absorbed by low energy electrons in connection with precision measurements of the cyclotron frequency of free electrons. These authors neglect the effect of collisions on the lineshape which is discussed in the final part of the present paper.

2. Electron–microwave interaction

The equation of motion of an electron (charge $-e$, rest mass m_0) moving in a magnetic field $\boldsymbol{B}$ (assumed to be in the z direction of a rectangular cartesian coordinate system

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and under the action of a microwave electric field $\mathscr{E}$ (assumed to be in the x–y plane) is

$$\dot{\boldsymbol{p}} = -e\left(\mathscr{E} + \frac{\boldsymbol{p}}{m_0\gamma} \times \boldsymbol{B}\right) \tag{1}$$

where $\boldsymbol{p}$ is the electron momentum and $\gamma = \{1+(p^2/m_0^2c^2)\}^{1/2}$. Although initially low electron energies are to be considered, equation (1) is written in a relativistic form since the interaction accelerates electrons to energies for which the relativistic effect becomes important. If complex notation is used for $\mathscr{E}$ and $\boldsymbol{p}$, that is,

$$\mathscr{E} = \mathscr{E}_x + \mathrm{j}\mathscr{E}_y$$

$$\boldsymbol{p} = p_x + \mathrm{j}p_y$$

equation (1) becomes

$$\dot{\boldsymbol{p}} = -e\mathscr{E} + \mathrm{j}\omega_c\boldsymbol{p} \tag{2}$$

where ω_c $(=eB/m)$ is the electron cyclotron frequency. If ω is the angular frequency of the microwave field then $\mathscr{E}$ is of the form

$$\mathscr{E} = \mathscr{E}_+\,\mathrm{e}^{\mathrm{j}\omega t} + \mathscr{E}_-\,\mathrm{e}^{-\mathrm{j}\omega t}, \tag{3}$$

that is, there will be anticlockwise (positive) and clockwise (negative) rotating components. For a circular polarization either $\mathscr{E}_+$ or $\mathscr{E}_- = 0$, whilst for transverse polarization $\mathscr{E}_+ = \mathscr{E}_-$.

In order to obtain analytical solutions giving the magnitude of the momentum (p) (and hence the energy of the electrons) it is convenient to write $\boldsymbol{p} = p\,\mathrm{e}^{\mathrm{j}\phi}$ where ϕ is the angle between $\boldsymbol{p}$ and some fixed direction in the x–y plane. After some algebraic manipulation of the previous equations we obtain

$$\dot{p} = -e\mathscr{E}_+\cos\chi - e\mathscr{E}_-\cos(\chi+2\omega t), \tag{4}$$

where $\chi = \phi - \omega t$ and is the phase angle between the momentum vector and the electric field.

Equation (4) shows that at near-resonance, that is when $|\omega-\omega_c| \ll \omega_c$, the average change of momentum is given by the first term; the presence of a counter-circulating electric field has no influence on physically interesting results considered over times large compared with a cyclotron period.

Further manipulation of the equations gives $\dot{\chi}$:

$$\dot{\chi} = \omega_c + \frac{e\mathscr{E}_+\sin\chi}{P} + \frac{e\mathscr{E}_-}{p}\sin(\chi+2\omega t) - \omega. \tag{5}$$

Thus, if we consider near-resonant conditions, the temporal behaviour of p and χ is described by

$$\dot{p} = -e\mathscr{E}_+\cos\chi \tag{6}$$

$$\dot{\chi} = \omega_c - \omega + \frac{e\mathscr{E}_+}{p}\sin\chi. \tag{7}$$

Equations (6) and (7) were solved numerically and p and χ were obtained as functions of time for chosen values of the variable parameters $\mathscr{E}_+$, B, etc for resonant conditions. The microwave frequency was chosen to be 9·3 GHz.

It is well known that an electron initially in synchronism with the microwave field, falls out of synchronism whilst undergoing cyclotron acceleration due to the relativistic mass and that this limits the energy attainable in such a process.

Under the conditions which normally prevail in the cyclotron resonance of free electrons it is assumed that an electron moves in a circular orbit of radius $r = p/Be$. This assumption is valid if any changes occurring in p do so at a rate very much lower than the cyclotron frequency. Alternatively, if the relative magnitudes of the forces acting upon the electron are considered, it is seen that the Lorentz force is greater than, equal to, or less than the electric force according to whether $v \gtreqless \mathscr{E}/B$. Thus only in those cases for which $v \gg \mathscr{E}/B$ will the electron move in a circular orbit and any deviation from such an orbit will increase as v decreases.

If a low energy electron encounters the microwave electric field at such a phase that deceleration takes place, a sudden change in phase angle ensues due to the deviation from a circular orbit. Figure 1 shows the variation in phase angle and energy for an electron,

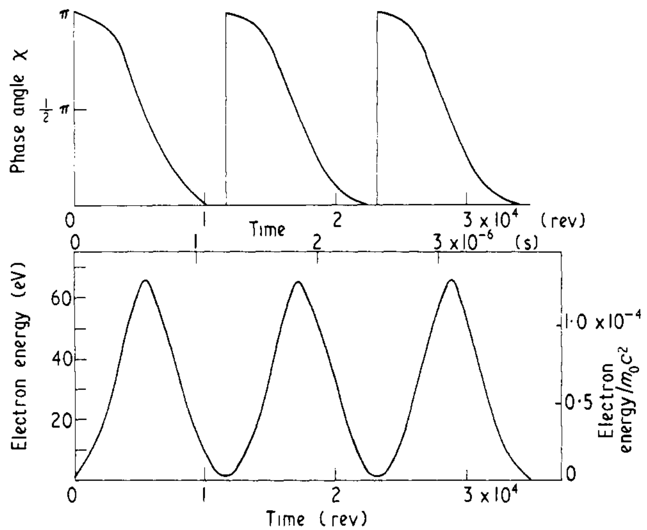

Figure 1. Variation in phase angle and energy for an electron, initial energy 1 eV, injected with a phase angle π. Microwave electric field = 10^2 V m^{-1}.

initial energy 1 eV, injected with a phase angle π with respect to a microwave electric field of 10^2 V m^{-1}. Figure 2 shows the phase angle as a function of time for three values of the ratio Lorentz force/electric force. It can be seen that when the electric force is appreciably greater than the Lorentz force the phase change takes place in a time comparable with the cyclotron period.

It is interesting to note that the period of energy oscillations for low energy electrons is very similar to the period of pendulum oscillations of small amplitude around a phase stable orbit for higher energy electrons originated by Bohm and Foldy (1946). E E Schneider (1972, private communication) has suggested that, by using the equations of motion in a dimensionless form, a relationship between the pendulum period and the period of low energy oscillations may be derived.

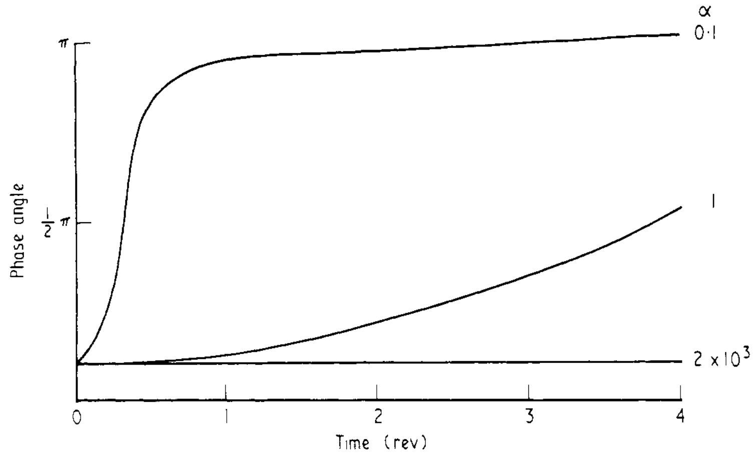

Figure 2. Phase angle as a function of time for different magnitudes of electric and magnetic forces. α is the ratio of the magnetic to the electric force.

3. Power absorption by low energy electrons during cyclotron resonance

In contrast to the last section, the calculations presented here deal with cases in which the microwave electric field strength is low enough to prevent substantial acceleration and consequent broadening of the absorption line. Using the same notation as in the previous section, the equation of motion of an electron interacting with a rotating electric field $\mathscr{E}\exp\{\mathrm{j}(\omega t+\phi)\}$ is:

$$\frac{\mathrm{d}v}{\mathrm{d}t}+\frac{v}{\tau}=\frac{e\mathscr{E}}{m}\exp\{\mathrm{j}(\omega t+\phi)\}+\mathrm{j}\omega_{\mathrm{c}}v \tag{8}$$

which has the solution:

$$v=v_0\exp\left\{\mathrm{j}\left(\omega_{\mathrm{c}}-\frac{1}{\tau}\right)t\right\}+\frac{\tau e\mathscr{E}}{m}\exp(\mathrm{j}\phi)\frac{\exp(\mathrm{j}\omega t)-\exp\{\mathrm{j}\omega_{\mathrm{c}}-(1/\tau)\}t}{1-\mathrm{j}(\omega-\omega_{\mathrm{c}})\tau}, \tag{9}$$

where τ, the mean time between collisions, is assumed to be independent of v. Furthermore it is assumed that the electron velocity is completely random immediately after each collision. The electron will absorb energy ΔE_{k} in a time T where

$$\Delta E_{\mathrm{k}}=E_{\mathrm{k}}-E_{\mathrm{k}}^0+\int_0^T\frac{mv^2}{\tau}\,\mathrm{d}t. \tag{10}$$

E_{k}^0, E_{k} are the kinetic energies at times 0 and T and the final term represents the work done against the frictional force. Using equations (9) and (10) gives

$$\Delta E_{\mathrm{k}}=\frac{T^2e^2\mathscr{E}^2}{m(a^2+X^2)}\left(1-\exp(-a)\cos X+a-\frac{\exp(-a)a(X\sin X-a\cos X)+a^2}{a^2+X^2}\right) \tag{11}$$

with $X=(\omega-\omega_{\mathrm{c}})T$ and $a=T/\tau$. In obtaining this expression the assumption that there is no correlation between ϕ and the time at which an electron enters the microwave field has been made. If $a\gg 1$, equation (11) reduces to a lorentzian form, similar to that considered by Schneider (1960). On the other hand, if $a\to 0$, equation (11) reduces to the $\sin^2(\frac{1}{2}X)/(\frac{1}{2}X)^2$ form quoted by Hsu and Robson (1965) and Franken and Liebes (1959).

The amplitude of the lineshape obtained in the absence of collisions in general decreases more rapidly in the wings than for the lorentzian line. Furthermore, secondary peaks occur in the former case. The full widths at half height for the resonance lineshapes obtained under these limiting conditions are:

$$\begin{array}{lll} \text{(i)} & \text{lorentzian} & \Delta\omega = \dfrac{2}{T} \\ \text{(ii)} & \dfrac{\sin^2\frac{1}{2}(\omega-\omega_c)T}{\{\frac{1}{2}(\omega-\omega_c)T\}^2} & \Delta\omega = \dfrac{2{\cdot}78}{T}. \end{array} \tag{12}$$

In addition to investigating the lineshapes for these two cases, intermediate cases may be considered if a new time parameter T_{eff} defined as

$$T_{eff} = \left(\frac{2{\cdot}78}{T} + \frac{1}{\tau}\right)^{-1} \tag{13}$$

is introduced and the X of equation (11) is replaced by

$$y = X\frac{T_{eff}}{T}. \tag{14}$$

The mean power absorbed is then

$$\frac{\Delta E_k}{T} = \frac{e^2\mathscr{E}^2}{m} bT_{eff}F(a)S(y, a) \tag{15}$$

where $b = (2{\cdot}78 + a)$; $F(a)$ is a normalizing factor given by

$$F(a) = \frac{a + \exp(-a) - 1}{a^2}$$

and $S(y, a)$ is a 'shape function' which does not depend on the absolute value of the mean time between collisions and transit time, but merely on the ratio of these quantities:

$$S(y, a) = \frac{1}{F(a)}(a^2 + b^2y^2)^{-1}[\![2\sin^2(\tfrac{1}{2}by) + \{1 - \exp(-a)\}\cos(by) + a + 2a^2(a^2 + b^2y^2)^{-1}[e^{-a}\{\cos(by) - \tfrac{1}{2}by\sin(by)\} - 1]]\!]. \tag{16}$$

$S(y, a)$ was computed as a function of y for several values of a and is shown in figure 3. It is seen that when the transit time is equal to or smaller than the mean time between collisions, secondary peaks occur in the wings of the resonance lines. Similar secondary peaks have been observed in the cyclotron resonance signals associated with 10 keV electrons (details to be published). It should be noted, however, that due to the relativistic increase in the electron mass with increasing energy, the profiles for medium energy electrons show regions of negative power absorption.

4. Conclusion

The equations of motion for a low energy electron undergoing cyclotron resonance were solved numerically enabling the variations in electron energy to be determined. The computer solutions of the equations of motion may prove useful in attempts to explain

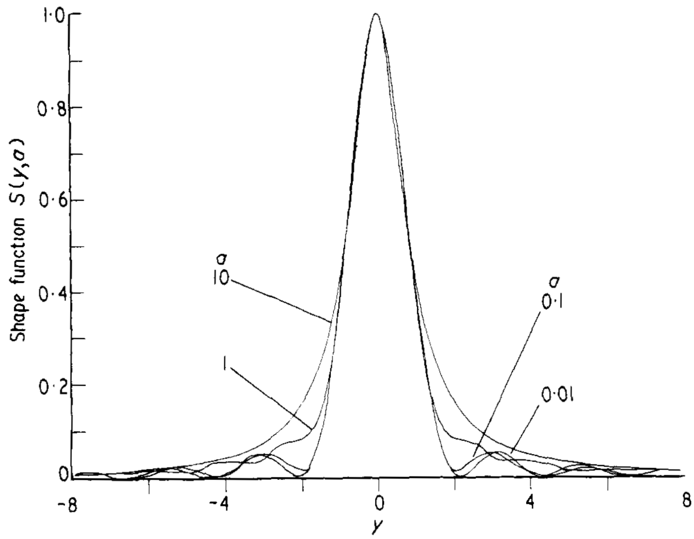

Figure 3. The shape function $S(y, a)$ as a function of

$$y = (\omega - \omega_c)\left(\frac{2{\cdot}78}{T} + \frac{1}{\tau}\right)^{-1}$$

for several values of $a(= T/\tau)$.

results from experimental investigations into electron temperatures in plasmas produced in electron cyclotron resonance heating guns (Masuda *et al* 1972).

The cyclotron resonance lineshapes in the absence of large microwave fields were calculated and the existence of secondary peaks similar to those observed in later experiments (to be published) was demonstrated.

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